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13 ABSTRACT (Maximum 200 words)

The main purpose of this research is to develop a rigorous mathematical framework for the design of control laws for feedback stabilization and for controllability of the transient behavior of flexible structures based on distributed parameter models of such structures. This work has entailed deriving accurate distributed parameter models for elastic structures and understanding the implications of the various models for the controllability and stabilizability of structures. Substantial progress has been made for models of multiple-link constructions that are composed of elastic beams, plates, shells or combinations of such elastic elements. Such structures are representative of trusses, frames, robot arms, solar panels, antennae, deformable mirrors, etc., currently in use.

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## Final Technical Report (AFOSR 88-0337)

September 1, 1988 - November 30, 1991

P.I. John E. Lagnese

The main purpose of the research undertaken under the above grant was to develop a rigorous mathematical framework for the design of control laws for feedback stabilization and for controllability of the transient behavior of flexible structures based on distributed parameter models of such structures. Thus the focus of the effort was, first, to derive accurate distributed parameter models for elastic structures and, second, to understand the implications of the various models for controllability and stabilizability of structures.

In order to carry out the program, attention was first given to the issues of controllability and stabilizability of particular structural elements such as beams and plates. More recently, substantial progress has been made towards developing models for networks of such elements, that is, multiple-link constructions that are comprised of elastic beams, plates, shells or combinations of such elastic elements. Such structures are representative of trusses, frames, robot arms, solar panels, antennae, deformable mirrors, etc., currently in use.

Listed below are several projects which were successfully completed under the grant. These are intended only to be representative of the work performed. The list of publications below will give a better idea of the scope of our effort.

**Exact controllability of thermoelastic plates.** In the design of control laws for elastic structures it is obviously important to understand the effects that thermal forces such as solar heating or cooling have on the control process. This issue was investigated in the context of a study of exactly controlling the motion of a thermoelastic plate. This problem is considerably more difficult than the analogous one for a purely elastic plate because of the dissipation which thermal forces may produce in the plate. Then the control forces must, in effect, work against thermal dissipation. This problem was first attacked in joint work with J.-L. Lions. However, the results obtained were not entirely satisfactory since (a) the controls were constrained to act in the Dirichlet boundary conditions, which cannot be easily implemented, and (b) the control time  $T$  obtained depended on a small thermal parameter  $\varepsilon$  and  $T(\varepsilon) \rightarrow \infty$  as  $\varepsilon \rightarrow 0$ , a conclusion entirely counterintuitive. However, in [1], the exact controllability problem was solved using boundary controls in edge forces and moments, which are physically implementable, and a control time independent of  $\varepsilon$  was established. This allowed the study to the behavior of the reachable set as  $\varepsilon \rightarrow 0$ , and the appropriate limiting behavior was established.

**Stabilization under nonlinear feedback.** Many, if not most, passive damping devices are of velocity feedback type in which a force or moment is applied

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that is some (generally unknown) function of the current velocity at some part of the structure. Therefore it is important to quantify the specific stabilizing effect that such a device will impart to the structure. In recent years, there have been many studies of this type in which it was assumed that the feedback mechanism is a *linear* function of velocity. However, a linear velocity feedback models an ideal damper, in particular, one not subject to internal friction, inertial forces, etc. Since no real damping mechanism responds linearly to velocity inputs over its entire operating range, it is important to study the dissipation induced by dampers which react to velocity inputs in a nonlinear fashion. Some initial results along this line were obtained by Zuazua (for wave equations) and by Lagnese (for plate equations). However, these works required the damping function to be differentiable when, in fact, internal friction manifests itself as singular (and possibly multivalued) behavior in the derivative at the origin. This restriction was lifted, in the context of the dynamic equations of plane elasticity, in [3], and in subsequent papers dealing with nonlinear beams and plates. In particular, the effect of the singularities in the feedback function on the decay rate was made explicit.

**Modelling and control of nonlinear structures.** This effort dealt with distributed parameter modelling, analysis and the design of control mechanisms for structural components (mainly beams and plates), and networks of such components, that are or may be subject to large displacements from their equilibrium states. Such phenomena are common in robotics, for example, because of the great flexibility of certain types of robot arms and the fact that rotations are superimposed on the longitudinal motion of the arm, and are certainly possible for any type of flexible structure under various types of loadings on the structure.

In large displacement situations there are complicated, nonlinear interactions between longitudinal and transverse displacements, shearing effects, torsion, etc. The resulting models are highly nonlinear and support a wide range of complicated dynamics including buckling and even fracture. It is important to mathematically analyze such models in order to develop an understanding of possible dynamical behavior and as a first step towards the development of mechanisms for their stabilization and control.

In this direction, in joint work with my colleague G. Leugering, the planar motions of a uniform prismatic beam were considered, and a nonlinear dynamical model that reflects the effects of stretching of the beam on its bending was derived. Even in the simplest of situations where shearing and torsion are ignored, the nonlinear model is quite complex, involving singularities and equations of changing type, among other confounding factors. Under some further simplifying assumptions, one obtains a beam model that is the analog of the von Karman plate model. In this particular case a complete analysis of the dynamics, the effects of various (possibly nonlinear) dissipative feedback mechanisms on the asymptotic

behavior of solutions, and an exact controllability theory were developed in the papers [4,7,8]. This work was extended to nonlinear plates in [11].

**Modelling and control of multiple-link structures.** This is the most recent part of our effort, beginning with the question of modelling the dynamic interactions at junctions between beams. Considerable progress has already been made in this direction, see [13], where a general three-dimensional model incorporating torsion, thermal effects, nonlinear effects, etc, has been presented. A study of controllability and stabilizability of planar networks of vibrating beams consisting of several Timoshenko beams connected to each other by rigid joints at all interior nodes of the system was given in [14]. It was assumed that some of the exterior nodes are either clamped or free; controls could be applied at the remaining exterior nodes and/or at interior joints in the form of forces and/or bending moments. Our goal was to determine whether, for a given configuration, it is at all possible to drive all vibrations to the rest configuration in a given finite time interval by means of controls acting at some or all of the available (non-clamped) nodes of the network and, if so, where such controls should be placed. Alternatively, a control objective is to construct energy absorbing boundary-feedback controls that will guarantee uniform energy decay. It was demonstrated that if such a network does not contain closed loops and if at most one of the exterior nodes is clamped, exact controllability and uniform stabilizability of the network is indeed possible by means of controls placed at the free exterior nodes of the system. On the other hand, examples were presented to demonstrate that when a closed loop is present in the network or if the network has more than one clamped exterior node, it may happen that approximate control of the network to its rest configuration is not possible even if controls are placed at every available node of the system. Although it was shown that this sort of behavior is anomalous, the fact that it may occur has important implications for the both the design of structures and their control laws.

#### **Research Papers and Books, 12/1/88–11/30/91**

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[10] *Local controllability of dynamic von Karman plates*, **Control and Cybernetics**, 19 (1990), pp. 155-168.

[11] *Modelling and Stabilization of nonlinear plates*, **Inter. Ser. Numer. Math.**, 100 (1991), pp. 247-264.

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[13] (with G. Leugering and G. Schmidt) *Modelling of dynamic networks of thin thermoelastic beams*, **J. Math. Meth. Appl. Sci.**, accepted for publication.

[14] (with G. Leugering and G. Schmidt) *Control of planar networks of Timoshenko beams*, **SIAM J. Control Opt.**, submitted.

[15] (with G. Leugering and G. Schmidt) *Modelling and controllability of networks of thin beams*, **Proc. 15th IFIP Conf. on System Modelling and Opt.**, to appear.

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